Evolution and Learning in Collective Decision Making

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Abstract

We develop a model of social interaction among individuals in the political market. We depart from the standard economic hypothesis by assuming a bounded individual rationality. The evolutionary character of the model is based on the process by which individuals take their political decisions: it tries to link decisions and outcomes through a learning process. To this extent we use concepts from the cognitive sciences and try to apply them, with a different degree of success, to public choice.

1 Introduction

The influence of James M. Buchanan's contributions to political economy has led to an ongoing and fruitful research program. Part of it has been focused on the process by which individuals coordinate leading to a given social order within a contractarian approach. A form of social cooperation that may shape the institutions of an economy, being the social order of a free society the ultimate goal of this work. In this ground, as Yeager [28] and Braid [28] point out, Buchanan's work has some common points with Austrian economics[1]. We need not insist on them as these will be familiar to readers: knowledge, discovery and disequilibrium, and of course subjetivism are some common aspects of a non-orthodox way of understanding

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[1]However there are also some points of conflict: in his *Limits of Liberty* [7], Buchanan poses his concerns about the limits of evolution. His criticism to the hayekian concept and its applicability to social institutions casts no doubts about his lack of enthusiasm for a social organization based upon such a concept. Anyhow it must be noted that his critic is directed towards the efficiency properties of evolution. Although we develop an evolutionary model of political decision making it does not embody the evolution of political institutions, that we consider given.

social phenomena. Much of this view can be characterized by the exchange paradigm. Exchange that implies a continuous process of interaction among individuals that lead to a knowledge accumulation and spread, and to the evolution of the social systems, and that differs from automatic application of restricted maximization to social problems.

Following this paradigm, this paper addresses to the learning process of individuals in the political market. We are interested in evolution and the disequilibrium properties of the exchange that takes place in the collective action. To this end we will resort to experimental simulation of a very simplified model of an economy. In doing so, we characterize individuals by their learning capabilities rather than by an extremely demanding concept of rationality. The paper is structured as follows: next section discusses the theoretical framework; then the results of some experiments are shown; the final section is devoted to discussion and conclusions.

2 Modeling learning in decision making in a political economy

We start this section by briefly describing the model on which the computer simulations were based. The outcomes of the model are the result of individual decisions in the collective realm. The basic setup as well as the learning process is partly based on a model by Lettau and Uhligh [18], but generalized for a context of collective decision making with N agents. In this, individuals identify the state of nature that in the model is given by the level of income. Taking this as a data, and analyzing past options and their influence on future states, individuals take next action as the result of evaluating their political options. A political option of an individual is a mapping from the set of states to the set of decisions. In this paper we are concerned with the evaluation of political options and the weak link between individual decisions and the outcomes of political processes. A question that Buchanan and Tullock [9] already stressed as a limit to individual rationality in the public sphere.

2.1 A basic model of learning

Assume an economy with N individuals. For our purposes we will consider a stationary population interacting during T periods. The main features of the model are:

1. At the beginning of each period t, every individual i is endowed with an amount of resources that will be denoted as y_t^i, and can be considered as period t income. Endowments follow a *Markov chain* with m different states. The probability of transition from state i to state j, given $i \neq j$, is:

$$p_{ij} = \begin{cases} \frac{(1-\rho)}{m} & i \neq j \\ \rho + \frac{(1-\rho)}{m} & i = j \end{cases} \quad (1)$$

where ρ plays the role of an autocorrelation term . The closer it is to one, the higher the probability of remaining at the same state. In this way we ensure an autoregression in the process. For simplicity we assume $m = 2$, leading to two possible situations for an individual: high income ($\overline{y}$) and low income($\underline{y}$).

In addition, while being at one state, income follows a random walk:

$$y_t^i = y_{t-1}^i + u_t^i \quad \text{where } u_t^i \sim NID(0, \sigma) \quad (2)$$

This representation allows for a dynamic evolution of the income with a stochastic trend.

2. Individuals derive utility from consumption. Thus in a world without public sector, and ruling out intertemporal transfers of income by means of wealth accumulation, an efficient allocation is one in which all income is consumed. Let q_t^i be the quantity of private good consumed. Then the utility of individual i is given by $u(q_t^i) = u(y_t^i)$[2].

3. As our work addresses to the allocation of resources between private activities and public ones, the next step is to include a public choice mechanism. After income has been determined, collective decision takes place. At that stage individuals vote for a political program. Two alternatives were considered for simulation purposes:

   (a) A situation in which individuals vote for a redistribution level. In this case the net utility is derived from its initial income plus (minus) the transfers from (towards) the public sector.

   (b) A situation in which individuals vote for a public supply of a pure public good, Q. Now individuals include another argument in his utility function, i.e. the total amount of public good provided ($u(q_t^i, Q_t)$).

   In both cases individuals choose a proportional tax system. We have considered two main scenarios for public choice to take place. One in which applying the majority rule, the median voter determines the outcome of the process; another in which individuals vote for political parties. Restrictions to the maximum level of redistribution/public good supply, as well as the decision making rules are considered to be constitutionally fixed.

[2]We used two utility functions in our model: a logarithmic transformation of a Cobb-Douglas function, and a CRRA function.

4. Once the amount of redistribution or public good supply has been chosen in the political process, an individual evaluates the *fitness* of his political option. In doing so, she considers the utility from consumption, the state of the world and the state her decision induces. At this point we have to mention that the fitness of an option has to account for the fact that there can be no direct link between an individuals choice and the political outcome. Additionally, as there may be resources invested in gathering information, voting may be costly. Both aspects will be considered. Finally this fitness leads to a ranking of options that will help the individual in taken decisions in the future.

The process of evaluating and updating the fitness of political options is an evolutionary one. Learning which options are most fitted for a given state is the main feature of this process. The mechanism by which learning takes place is briefly discussed next.

2.2 From learning to evolution

The main point of this paper is that political decisions taken by individuals are the result of evolution and learning. This need not exclude rational behavior, as it is also possible to learn to be rational. However not all decisions will be rational in the economic sense of the word, but may be rational in a broader sense. Given the set of all the possible states of nature a political option[3] is to be understood as a mapping from a given state into an action or decision. The most basic action in a public choice setting is voting. To take a political option and vote accordingly implies consider the implications of that decision. To some extent whenever individuals are in the public domain their decisions are affected by decisions they took in the past and by the outcomes of political processes. This simple idea may be formally developed by using the concept of a *classifier system.*

We will begin with the intuition behind it. Given an state (in our example high income/low income) an individual has to decide the allocation of his resources between private or public consumption[4]. The effect of a private decision is the utility of consumption. Public decisions are converted into political outcomes through the political process; these may affect the utility of individuals either increasing or decreasing it. Define the *fitness* of a political option as an ordinal value that the individual assigns to an action given a state. Define a classifier as the pair of an option with its related fitness. Then a classifier system is the set of all possible pairs $\langle$option, fitness$\rangle$.

[3] A political option in public choice is the counterpart of a rule in the private realm. Although they have the same meaning, in collective decisions individuals follow options rather than rules.

[4] As there are differences between redistribution and public good consumption, in this example we consider a negative income tax schedule.

The problem we are considering can be stated in the following terms: given a state of nature, an individual has to choose a political option. The logic underlying the actual setup is that an individual will choose the option which is most fitted with the actual state. The fitness of an option in a classifier system is given by the utility of the individual and the state it induces. Hence there is a discounting of the effects of an option in the future.

The obvious question is how an individual links an option with its fitness. As the outcomes of political processes may have little to do with the option an individual took, there may be no way in which an individual updates the utility derived from a political situation. This is in fact the main problem that we faced in our example but we tried to solve in a simple way. We will illustrate this with an example.

Consider the median voter of a political process in which decisions are taken by using majority rule. In this case her options will be the winning ones. This means that there will be a direct link between option and political outcomes. In this case the updated fitness of option k, that was taken at period $t-1$, in period t will be:

$$\Delta r_t^k = f(r_t^j, r_{t-1}^k, u(q_{t-1})). \tag{3}$$

Expression (3) updates the fitness of an option by using the discounted fitness of the option that it may induce in t, r_t^j, the utility of that option $u(q_{t-1})$ (now as we are in the median voter case this utility points towards the political process), and its own fitness[5]. However not all individuals are the median voter and not all political decisions use the majority rule. This means that options may not be translated into individually desirable outcomes. For this case an alternative updating function is proposed in which the outcome in t is taken into account

$$\Delta r_t^k = f(r_t^j, r_{t-1}^i, r_{t-1}^k, u(q_{t-1})). \tag{4}$$

Here option i refers to the outcome of the political process, and option k to the chosen option which need not coincide. Finally equation (4) can be generalized to include the cost of political decision making. If we assumed that there are costs in the act of voting then we have expression (5):

$$\Delta r_t^k = f\left(r_t^j, r_{t-1}^k, u(q_{t-1}), c_{t-1}\right), \tag{5}$$

[5]Expression $f(\cdot)$ in equations (3), (4) and (5) may be interpreted as a dynamic error correcting mechanism. It is usual to find this term as a convergent sequence over time by adding a decreasing sequence. This diminishes the weight of the correction term over time. In our simulations we employed the sequence:

$$\omega \frac{1}{t} f(\cdot).$$

where c_{t-1} represents the disutility from voting. In this last expression c amounts not only to the resources allocated in the political decision making process, but also for the gap between political option and outcomes.

To summarize, individuals evaluate their political options (and their related decisions) in terms of their relative fitness for a given state. This fitness is evaluated and updated according to past information but also discounts, although in a limited way, the future. It is from this evolutionary process that political outcomes emerge. In the next section we will draw the conclusions from different computational experiments.

3 Results from computational experiments

Three different simulation experiments were conducted by using the basic model described in the previous section. To this end we simulated a political economy with a stationary population of $N = 1000$ individuals. These individuals interacted during $T = 1000$ time periods. At the beginning of each interaction the endowment of each individual was fixed, and hence the state, according to expressions (1) and (2). With respect to income there are two additional facts to be considered. First, the ratio between high income/low income has been considered fixed and equal to $\overline{y}/\underline{y} = 4$. Second, in order to seed the initial population we set parameter P_0, which is the probability of being in the low income group at $t = 0$. Both data will appear in most of the results table.

Then, political decisions are taken among the different available alternatives. To this end, three different political scenarios were considered. The first one describes a political process in which decisions are directly taken by a simple majority rule; this leads to the well known median voter theorem results. The second one changes to a representative democracy in which three parties offering three different programs compete for votes. In this case the winning party defines the political action to be taken. Finally, the third one is a generalization of the previous one which includes the option of non-participation in the political process.

Individuals vote for a proportional tax rate t_j^i. Given individual options and a political setting, the outcome of the process is t^*. The total amount of taxes raised, $t^* \sum y_t^i$, will be collectively used either for providing a quantity Q_t of public good, or for redistributive purposes. In the former case all individuals in the economy equally benefit from the consumption of the public good. In the latter case a negative income tax (NIT) system is proposed, such that:

$$T_t^i = -S + t^* y_t^i \tag{6}$$

being $S = Q/N$. Once political options are evaluated and decisions emerge, and the political outcome is known, voters update their classifier systems and the process begins again.

				Fraction of voters	
P_0	ρ	Range of t^*		Below t^*	Above t^*
0.50	0.95	0.3818	0.3111	0.4335	0.5245
0.75	0.95	0.3888	0.3181	0.4506	0.5493
0.25	0.95	0.3959	0.3181	0.4474	0.5609
0.50	0.50	0.3888	0.3111	0.4337	0.5444

Table 1: Provision of a pure public good. The median voter theorem case.

				Fraction of voters	
P_0	ρ	Range of t^*		Below t^*	Above t^*
0.50	0.95	0.3888	0.3252	0.4530	0.5567
0.75	0.95	0.3818	0.3181	0.4518	0.5272
0.25	0.95	0.3818	0.3181	0.4452	0.5328
0.50	0.50	0.3818	0.3252	0.4524	0.5334

Table 2: Redistribution through a negative income tax. The median voter theorem case.

3.1 Applying the median voter theorem

In this section we discuss the main simulation results of the model considering that:

- voters are distributed along a line of political options;
- decisions are taken by means of majority rule.

This leads to a direct application of the median voter theorem and its well known results. Tax rates are restricted to be in the range $[0.0, 0.7]$. Table 1 presents the main results when the outcome of collective action is the provision of a public good. Table 2 shows these results in the NIT case. Both show very similar outcomes for different initial probabilities of being in a *low income* state P_0. Outcomes are the average of 1000 runs of the program.

It can be seen that the amount of redistribution/public good provision is located in all cases in a central range of the tax spectrum. Whenever this tax spectrum was modified the results were modified consistently[6]. Finally the population was almost equally distributed around the median result for different population compositions.

[6]In fact we run a simulation with a narrower range of tax rates $(0.0 - 0.3)$. Median results ranged from $0.13 - 1.166$; these directly points to our conclusions.

3.2 A representative democracy with political parties

Consider now a representative democracy with three different political parties: R, C and L. R offers a low level of public redistribution; C offers a higher collectivization of private income; finally L offers the highest level. For simulation purposes we set the tax rate of each party as 0.1, 0.35 and 0.65 respectively. Now, individuals cast their votes for a party, and the winner decides the policy[7].

Table 3 and 4 show the results of this simulations. Again outcomes are an average of the total number of simulations, so they will give a rough view of the evolution of the process. If we focus in the redistribution, table 3, now it seems that the initial income distribution of the population matters. The first row of the table shows the situation in which low income individuals are approximately half the population. In this case, chances are that a high redistribution of income may be a fitted option for that half of the population; the ordering of the three programs point to this fact. However if the proportion of low income individuals is over 50% (for example an 85%, in the second row), redistribution will tend to be lower, as there will be low income individuals which may not benefit from this redistributive process. The same conclusion, but for the opposite reason, is drawn when only 15% of the initial population belongs to the low income individuals. Finally, the fourth row shows the case in which $\rho = 0.5$. This means that there is a 50% probability that an individual changes from one state to another. A moderate result is the outcome of this simulation. This shows how individuals do, partially, learn from past experiences: they can be either net contributors or receivers of the tax system.

If we change to the provision of a public good (see table 4), most results of the previous experiment hold. However these cannot be interpreted in the same way. As long as now all individuals are net receivers of the public good, there should be a trend towards more extreme results. This shows that individual rationality is bounded.

3.3 The vote motive

One of the most controversial issues in public choice has been on the rationale of voting. As the expected effect of an individual vote is marginal, and as the costs of being rationally informed surpass the benefits, there are no rational motives for voting. However empirical data show that in real world democracies too many people vote; in this case they may not be taking rational decisions.

[7]This could be the case of many European representative democracies, where the majority of a parliament need not be supported by a majority of the votes of the population. For example in Spain the representation system tries to favor parties with a higher number of votes in order to reduce fragmentation.

		Fraction of wins		
P_0	ρ	R	C	L
0.50	0.95	0.1901	0.2252	0.5845
0.85	0.95	0.2302	0.5025	0.2672
0.15	0.95	0.3111	0.3493	0.3393
0.50	0.50	0.2032	0.4714	0.3253

Table 3: Redistribution through a negative income tax. Choosing among three options.

		Fraction of wins		
P_0	ρ	R	C	L
0.50	0.95	0.2252	0.2832	0.4914
0.85	0.95	0.5475	0.2572	0.1951
0.15	0.95	0.0610	0.6256	0.3133
0.50	0.50	0.3773	0.2012	0.4214

Table 4: Provision of a pure public good. Choosing among three options.

In order to analyze the possibility of non-participation (A), we have conducted a third simulation in which the act of voting was costly. Table 5 shows the results. In these, we considered both $\rho = 0.95$ and $P_0 = 0.5$ fixed. The cost of voting was introduced in three different ways.

1. First, as a fixed proportion of the income. In this case two additional options were considered: voting is costly (I), and voting to a non winning program is costly (II). In both cases results show that the number of abstentions was in average over one quarter of the population. Figure 1 show the evolution of 20 iterations of the population. It can be seen that the levels of non-participation may be significantly over the average, and that the three options are quite close. However the L program wins almost one out of two elections.

2. Second, as a fixed cost. Again two options (I and II) were introduced. Results show that option I is consistent with the previous results. However option II gave a highly percentage of L wins. In the former, levels of non-participation were significantly above the previous results.

3. Finally, it was considered that the results of public action may be distributed among the voters of a program. This is what we have called private appropriation of public programs[8]. By this we mean that the beneficiaries of public programs are the voters of the party.

[8]In the table appears as appropriation

	Fraction of wins			
Costs	R	C	L	A
Proportional to income I	0.2252	0.2832	0.4914	0.2883
Proportional to income II	0.2612	0.3213	0.4174	0.2501
Fixed cost I	0.2342	0.3543	0.4114	0.4602
Fixed cost II	0.2252	0.0860	0.6886	0.2562
Appropriation of public programs	0.0000	0.0000	1.0000	0.0567

Table 5: Redistribution through a negative income tax. Introducing costs.

In this case the cost of voting is introduced via the opportunity cost of not being in the winning party. Results are quite appealing, and the first 20 iterations may be seen in figure 2. Obviously individuals learn quickly and vote for the winning option (L). While most of them could be better off by voting a moderate alternative, approximately the 50% of the population, if they are not in a winning coalition, they will incur in higher costs. This leads to a kind of social dilemma. According to these results abstention falls to a low 5% of the population, proving that individuals are not rational but they try learn how to be.

If we ignore the conclusions drawn from the third option, the introduction of the cost of voting in a evolutionary model of a political process, leads to a middle point between public choice and political science. Individuals do vote more often than public choice predicts, but less often than political science considers they should. This may in part be explained by one reason: an individual effect on the final outcome of a collective decision is directly related to the degree of participation. If it is high, her probability of affecting the outcomes decrease. And conversely it increases for lower participation. Figure 1 shows this. The cycling in non-participation may be partly due to the process of discovering the varying marginal effect of individuals in elections.

4 Discussion and conclusions

The previous pages showed the results on the simulation of a very simplified experimental political economy. This way of modeling differs in a significant way from orthodox economic theory were rational individuals interact maximizing some well defined objective function. However if this approach is of limited scope in economics, it has more drawbacks in the political exchange where the means and objectives of individuals are fuzzily defined. Moreover, individuals do not posses an invariable view of the real world but it changes over time as they learn from situations they experienced and revise their previous views.

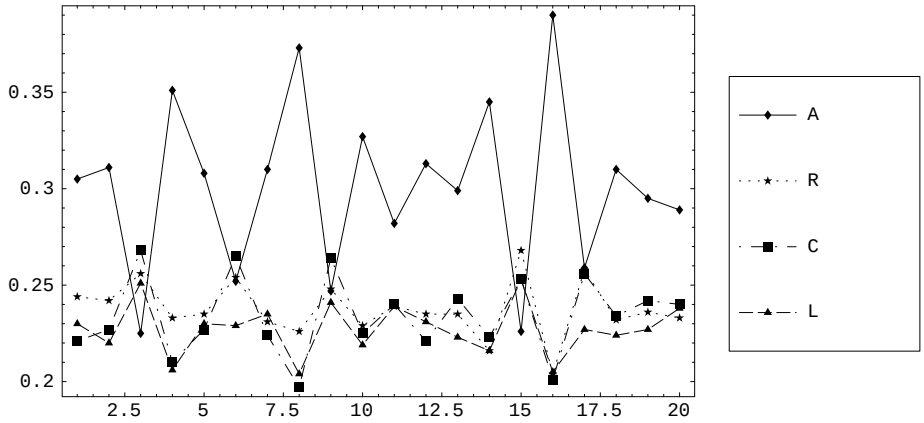

Figure 1: Dynamic evolution of votes and abstentions with proportional cost of voting.

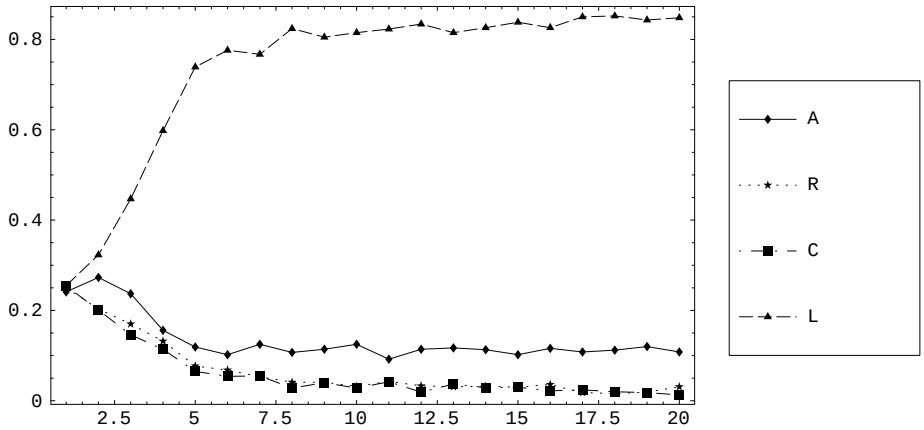

Figure 2: Dynamic evolution of votes and abstentions when public benefits are privately appropriated.

The evolutionary approach, while keeping some features of the orthodox models give more insights into the questions related to public choice and specifically collective decision making. While *irrational* social outcomes may arise from rational individuals, it is more likely that these have the origin in a limited concept of rationality. Individuals need not compute all the information an economist will suppose them to do, but just a limited set. Public choice has referred to this question as the *rational ignorance*, adding a link between public choice and the evolutionary literature. Using this new paradigm individuals may try to adapt and learn to the environment, perhaps searching for an optimum, but without explicitly optimizing. They just compile past information and return their actions as a direct consequence of this process. To some extent this is to suppose that individuals have different representations of the world (for our purposes the world is limited to the political process) that depend on their initial information endowments (that in our model were randomly generated), the situations and states they face, and the relation between decisions and outcomes. This implies that even "rational" choices will be different for different individuals[9]; then, why should limited rational behavior define a clear pattern?

The results of the simulations gave some interesting results:

- Constitutional restrictions do matter: the rule for making decisions, or the degree of collectivization, to mention two of the experiments, affects the outcome of the political process.

- We do not have to assume instrumental rationality to get rational results. When running the experiment on privatizing the benefits of public action, it was clear that almost all individual enroll the L program. Better be with them! If the social dilemma may be reproduced in experiments, this may help in detecting the origins and/or the solutions.

- However not all actions mimic rational behavior. In our setting, the degree of public provision of a pure public good was below the optimal level. In this case individuals did not achieved the standard of rationality that the neoclassical theory would suggest.

To conclude, it is not clear whether a collective decision making process may be better depicted by this kind of models or by a neoclassical setting. In any case it is a question under research that cannot be answered, at least in a definitive way, at this stage.

[9] As North [22] puts it, in a rational environment restrictions are unnecessary.

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